Name: Allen Xu **Math 8H 2025 Lesson 1 Basic Operations** Date: Sep. 4th

1. Indicate whether if the following statements are TRUE or FALSE:

$a-b=a+(-b)$ True	$-a+b=b-a=b+(-a)=b-a$ True	$a-(-b)=a+b$ True
$-a-(-b)=\cancel{b+a}\ -a+b$ False	$b-a=-(a-b)$ $+10-5=-(+5-10)$ $+5 \quad -(-5)=+5$ True	$a+b-c+(-d)=a+b-c-d$ True

2. Evaluate each of the following by adding or subtracting

a) $1+(-2)-3+4-(-5)$ $=1+(-2)+(-3)+4+5$ $=1+4+5+(-2)+(-3)$ $=1+4+5+(-5)$ $=1+4+5-5$ $=5$	b) $12-(-15)+(-20)$ $=12+15+(-20)$ $=12+15-20$ $=27-20$ $=7$	c) $8+(-12)-(-13)$ $=8+(-12)+13$ $=8+13-12$ $=21-12$ $=9$
d) $8+(-9)-(-11)-12$ $=8+(-9)+11+(-12)$ $=8+11+(-9)+(-12)$ $=19+(-21)$ $=19-21$ $=-2$	e) $-7+6-13+(-14)-(-23)$ $=-7+6+(-13)+(-14)+23$ $=6+23+(-7)+(-13)+(-14)$ $=29+(-34)$ $=29-34$ $=5$	f) $-20-(-15)+19-23$ $=-20+15+19+(-23)$ $=15+19+(-20)+(-23)$ $=34+(-43)$ $=34-43$ $=11$
g) $12+22-43+41-(-15)$ $=12+22+(-43)+41+15$ $=12+22+41+15+(-43)$ $=34+56-43$ $=90-43$ $=47$	h) $-12+(-13)-14+(-15)-(-16)$ $=-12+(-13)+(-14)+(-15)+16$ $=16+(-12)+(-13)+(-14)+(-15)$ $=16+(-54)$ $=16-54$ $=-38$	i) $-15+(-17)-19+23-(-22)$ $=-15+(-17)+(-19)+23+22$ $=23+22+(-15)+(-17)+(-19)$ $=45+(-51)$ $=45-51$ $=-6$

3. Evaluate each of the following the order of operations

a) $8+2\times5$ $=8+10$ $=18$	b) $9+3\times4\div2-3$ $=9+12\div2-3$ $=9+6-3$	c) $2+3\times4-6\div2$ $=2+12-3$ $=14-3$ $=11$
d) $4+3\times5-6\div2$ $=4+15-3$ $=19-3$ $=16$	e) $8\times(-4)+12\div(6)$ $=-32+2$ $=-30$	f) $1200\div2\times10\div5\div3$ $=\frac{1200\times\cancel{10}}{\cancel{2}\times\cancel{5}\times3}$ $=\frac{1200}{3}$ $=400$

7. Given that $(x-y)^2 = x^2 - 2xy + y^2$, what is the value of $997^2 - 2(997)(995) + (995)^2$

$x^2 \quad - \quad 2xy \quad + \quad y^2$

$x = 997 \quad y = 995 \quad (x-y)^2 = (997-995)^2$

$(997-995)^2$

$= 2^2$

$= 4$

8. Dave wrote 12 tests and got an average of 68%. If he gets 85% on every new test that he writes, how many more tests will he need to until he gets an average greater than 75%?

$68\% \times 12 = 816\% \quad 816\% + 85\% = 901\% \quad 901\% \div 13 \approx 69.3$

$901\% + 68\% = 979\% \quad 979\% \div 14 \approx 69.929$

$979\% + 68\% = 1047\% \quad 1047\% \div 15 = 69.8$

$1047\% + 68\% \times 6 \div 21 = 69.\overline{285714}$

Method 2!: (The smart way)

$68(12) + 85x = 75(12 + x)$

$\begin{array}{r} 68 \\ \times 12 \\ \hline 136 \\ 68 \\ \hline 816 \end{array}$

$\begin{array}{r} 69.3 \\ 13\overline{)901.0} \\ 78 \\ \hline 121 \\ 117 \\ \hline 40 \\ 39 \end{array}$

9. Mt. Everest, the highest elevation in Asia, is 29,028 feet above sea level. The Dead Sea, the lowest elevation, is 1,312 feet below sea level. What is the difference between these two elevations?

$29028 - (-1312)$

$= 29028 + 1312$

$= 30340$

$\begin{array}{r} 29028 \\ +\ 1312 \\ \hline 30340 \end{array}$

10. The sum of 8 consecutive numbers is 188, what are the numbers?

$188 \div 8 = 23.5 \quad 20 \ 21 \ 22 \ 23 | 24 \ 25 \ 26 \ 27$

23.5

The numbers are 20, 21, 22, 23, 24, 25, 26, and 27.

11. There are 12 sedans and 18 minivans. Each sedan has 5 females and each minivan has 7 males. What is the difference in the number of males and females?

$18 \times 7 - 12 \times 5$

$= 126 - 60$

$= 66$

12. Each burger at a fastfood chain costs $4.50 and each hotdog costs $2.25. If Dave spent $100 only on hotdogs and burgers, and has twice as many hotdogs than burgers, how many of each did he purchase?

$\$100 \div (\$2.25 \times 2 + \$4.50)$

$= \$100 \div (\$4.50 + \$4.50)$

$= \$100 \div (\$9.00)$

$= 11.1$ 11 Groups = 11 Bur & 22 Hot

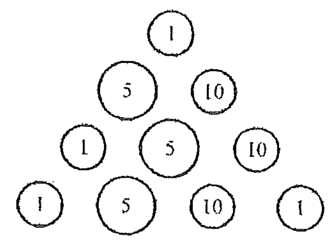
13. There are 90 students in a class and there are twice as many females and than males. The average score of the females in the class is 85% and the average score of the males is 70%. What is the average score of all the students in the class?

$90 \div (2+1)$ $\quad 30 \times 2 = 60$ females $\quad 60 \times 85\% = 5100\%$ $\quad 5100\% + 2100\% = 7200\%$

$= 90 \div 3$ $\quad 30 \times 1 = 30$ males $\quad 30 \times 70\% = 2100\%$ $\quad 7200\% \div 90 = 80\%$

$= 30$

14. Micah placed pennies, nickels, and dimes in rows according to the diagram so that each row contains one more coin than the previous. What is the number of cents in the value of all the coins in the first 13 rows?



Num of coins $= (1+13) \times 13 \div 2$

$= 14 \times 13 \div 2$

$= 7 \times 13$

$= 91$

$1 + 5 + 10 = 16$ cents

$91 \div 3 = 30 \cdots 1$

$30 \times 16 + 1$

$= 480 + 1$

$= 481$ cents

Name: Allen Xu Period 7 Date: Sep. 6th

Math 8H 2025 Lesson 2 Perfect Squares, Cubes, and Square Roots

1. Indicate which of the following numbers are perfect squares, cubes, both, or neither. If it is a perfect square or cube, write it as a cube or square:

a) 225 Perfect Square 15^2	b) 1024 32^2 Perfect Square	c) 243 Neither	d) 196 Perfect Square 14^2	e) 400 Perfect Square 20^2
f) 128 Neither	g) -1 Perfect Cube -1^3	h) 8000 Perfect Cube 20^3	i) 289 -17^2 Perfect Square	j) 125 Perfect Cube 5^3
k) 343 Perfect Cube 7^3	L) -8 Perfect Cube -2^3	M) 10,000 100^2 Perfect Square	n) 64 Both 8^2 4^3	o) 189 Neither, but $17^2=289$
p) 800 Neither	q) 729 9^3 Perfect Cube	r) 0 Both 0^2 0^3	s) 625 25^2 Perfect Square	t) 1331 11^3 Perfect Cube

2. Given each of the equations below, indicate whether if it is TRUE or FALSE, explain your work:

i) $\sqrt{-9}=3$ TRUE or FALSE
Can't be a square

ii) $\sqrt[3]{-64}=4$ TRUE or FALSE
$\sqrt[3]{-64}=-4$

ii) If the square of "A" is equal to "B", then the square root of "B" is equal to "A" : TRUE or FALSE $2^2=4$ $\sqrt{4}=2$

iii) A number can only be a perfect square or a perfect cube, but not both: TRUE or FALSE 64

iv) The square root of a negative number does not exist: TRUE or FALSE $N \times N = P$

v) The cube root of a negative number does not exist: TRUE or FALSE $N \times N \times N = N$
P

vi) Perfect squares can only be positive: TRUE or FALSE

vii) Perfect cubes can only be positive: TRUE or FALSE

viii) Suppose "a" is an integer and not a perfect square, then a^2 must be a perfect square TRUE or FALSE

ix) If "a" is a negative number then it can never be a perfect cube TRUE or FALSE

x) If "a" and "b" are positive integers and are NOT perfect squares, then $a \times b$ can be a perfect square TRUE or FALSE

xi) Suppose "a" and "b" are prime numbers, then $a \times b$ can never be a perfect square TRUE or FALSE
They can be the same numbers.

xii) Suppose "a" and "b" are not perfect squares, then $a \times b$ can never be a perfect square TRUE or FALSE

3. Draw a Number Line and Estimate each of the following

a) $\sqrt{50}$ $7^2=49$ $8^2=64$ 7 $\sqrt{50}$ … 8	b) $\sqrt{180}$ $13^2=169$ $14^2=196$ 13 $\sqrt{180}$ 14	c) $\sqrt{77}$ $8^2=64$ $9^2=81$ 8 $\sqrt{77}$ 9
d) $\sqrt{134}$ $11^2=121$ $12^2=144$ 11 $\sqrt{134}$ 12	e) $\sqrt{200}$ $14^2=196$ $15^2=225$ 14 $\sqrt{200}$ 15	f) $\sqrt{63{,}859{,}102}$ $\approx 64{,}000{,}000 = 8000^2$ 7990 $\sqrt{63859102}$ 8000
g) $\sqrt{0.0000485}$ $0.007^2=0.000049$ 0.0069 $\sqrt{0.0000485}$ 0.007	h) $\sqrt{2385029}$ $1500^2=2250000$ $1600^2=2560000$ 1500 $\sqrt{2385029}$ 1600	i) $\sqrt{0.0023501}$ $0.05^2=0.0025000$ 0.04 $\sqrt{0.0023501}$ 0.05

4. Suppose "a" is a perfect square, what numbers can the units digit be?

0, 1, 4, 9   5, 9

5. Suppose "A" and "B" are single digit positive integers, which of the following can be a Perfect Square?

i) $7ABC4$ ii) $8ABC2$ iii) $9ABC6$ iv) $75ABC44$

Yes   No   Yes   Yes

$1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$ $6^2=36$ $7^2=49$ $8^2=64$ $9^2=81$ $10^2=100$

No 2, 3, 7, 8

6. A square has a perimeter of 28cm. What is the area of the square in cm²?

$28 \div 4 = 7$cm $7^2 = 49$cm²

7. Two squares, each with an area of 30cm², are placed side by side to form a rectangle. What is the perimeter of this rectangle? Give your answer to 3 decimal places:

$\sqrt{30} \approx 5.4772$cm $5.4772 \times 6 = 32.863$

8. A cube has a volume of 125cm³. What is the area of one face of the cube?

$\sqrt[3]{125} = 5$cm

$5^2 = 25$cm²

9. What is the “RULE of 5’s”? What is the trick to squaring a number that ends with 5? Ie: 125 x 125 = ?

125 × 125 = 10(12) × [10(12) + 10] + 25 = 120 × 130 + 25

10. Suppose “A” is a single digit positive integer, what is the value of $A5 \times A5$ in terms of “A” ?

10 × A × [10(A) + 10] + 25

11. Square root the following without using a calculator:

a) $\sqrt{15625}$ = 125	b) $\sqrt{42025}$ 205	c) $\sqrt{93025}$ 305
d) $\sqrt{65025}$ 255	e) $\sqrt{497025}$ 705	f) $\sqrt{46225}$ 215

12. Which is bigger? 100^2 or 50^3 Explain your answer:

50^3, because $100^2 = 100 \times 100 = 50 \times 2 \times 50 \times 2 = 50 \times 50 \times 4$, but $50^3 = 50 \times 50 \times 50$. $4 < 50$, so 50^3 is bigger.

13. In the following equations, the letters a, b and c represent different numbers.

$1^3 = 1$

$\sqrt[3]{8} = 2$ $a^3 = 1 + 7$

27 $3^3 = 1 + 7 + b$ 19

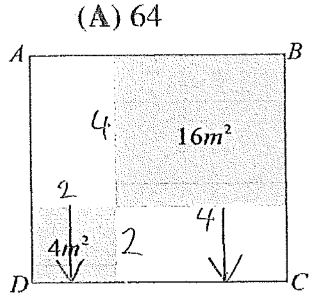
64 $4^3 = 1 + 7 + c$ 56

The numerical value of $a + b + c$ is $2 + 19 + 56 = 77$

(A) 58 (B) 110 (C) 75 (D) 77 (E) 79

14. $ABCD$ is a square that is made up of two identical rectangles and two squares of area 4 cm^2 and 16 cm^2. What is the area, in cm^2, of the square $ABCD$?

(A) 64 (B) 49 (C) 25 (D) 36 (E) 20

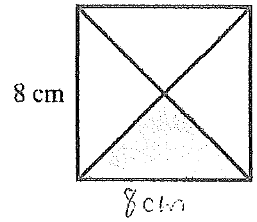


$(2+4) \times (2+4) = 36 \text{ cm}^2$

15. The diagonals have been drawn in the square shown. The area of the shaded region of the square is

(A) 4 cm^2 (B) 8 cm^2 (C) 16 cm^2

(D) 56 cm^2 (E) 64 cm^2



$8^2 = 64 cm^2$

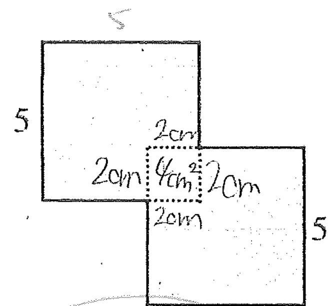
$64 \div 4 = 16 cm^2$

16.

Two squares, each with side length 5 cm, overlap as shown. The shape of their overlap is a square, which has an area of 4 cm^2. What is the perimeter, in centimetres, of the shaded figure?

(A) 24 (B) 32 (C) 40

(D) 42 (E) 50



$P = 5(4) + 3(4)$

$= 32$

$5^2 + 5^2 - 2\times4$

$= 25 + 25 - 8$

$= 42 cm$

17. Given that $a^2 - b^2 = (a+b)(a-b)$, what is the value of $1000^2 - 999^2$?

$a = 1000 \quad b = 999$

$(1000+999) \times (1000-999)$

$= 1999 \times 1$

$= 1999$

18. If $(k+3)(k-3) = 1000$, then what is the value of k^2 ?

$(k+3)(k-3) = k^2 - 3^2 = 1000$

$k^2 - 3^2 = 1000$

$k^2 = 1000 + 3^2$

$k^2 = 1000 + 9$

$k^2 = 1009$

$= 11$

Name: Allen Xu Period 7 Date: Sep. 12th

Math 8H 2025 Lesson 3 Multiplication Strategies:

1. When multiplying the following numbers together, which integers should you combine together first?

$2\times3\times4\times5\times7\times9$ (2×5)×4×(3×7×9)=10×4×(21×9)=10×4×189 =10×756=7560

2. When multiplying with a two digit number like 27, how should you split it into two values to make it easier to multiply? Explain:

Split into 20 and 7, because 20 is easy to multiply, and 7 is a single digit number which is also easy

3. Indicate how you would use the AREA Model to multiply the following:

a) 13×7 =10(7)+3(7) =70+21 =91	b) 21×8 =20(8)+1(8) =160+8 =168	c) 18×9 =10(9)+8(9) =90+72 =162 also: 18×9 =18(10)−18(1) =180−18 =162 rule of 9's
d) 24×7 =20(7)+4(7) =140+28 =168	e) 91×8 =90(8)+1(8) =720+8 =728	f) 135×8 =100(8)+30(8)+5(8) =800+240+40 =1080
g) 123×6 =100(6)+20(6)+3(6) =600+120+18 =738	h) 233×7 =200(7)+30(7)+3(7) =1400+210+21 =1631	i) 413×8 =400(8)+10(8)+3(8) =3200+80+24 =3304

4. Multiply the following by breaking it down and then combining factors:

a) 25×14 =5×5×2×7 =5×10×7 =35×10 =350	b) $2\times5\times3\times7$ =10×21 =210	c) $2\times3\times4\times10$ =6×4×10 =24×10 =240	d) 24×15 =4×6×3×5 =4×3×6×5 =12×30 =360	e) 35×6 =7×5×6 =7×30 =210
f) 45×8 =5×9×8 =40×9 =360	g) 28×12 =4×7×12 =4×84 =336	h) $25\times7\times4$ =25×4×7 =100×7 =700	i) $85\times3\times6$ =17×5×3×6 =51×30 =1530	j) $13\times11\times5$ =13×5×11 =65×11 =650+65 =715
k) $11\times17\times2$ =11×34 =340+34 =374	L) $123\times11\times3$ =123×3×11 =369×11 =3690+369 =4059	m) 5342×11 =53420+5342 =58762	n) $35\times12\times22$ =7×5×3×4×2×11 =7×3×5×2×4×11 =21×10×4×11 =210×4×11 =840×11 =8400+840 =9240	o) $77\times32\times25$ =7×11×4×8×25 =7×8×4×25×11 =56×100×11 =5600×11 =56000+5600 =61600

5. Use the formula $(a+b)(a-b)=a^2-b^2$ to evaluate each of the following

a) 17×15 $=16^2-1^2$ $=256-1$ $=255$	b) 19×21 $=20^2-1^2$ $=400-1$ $=399$	c) 14×16 $=15^2-1^2$ $=225-1$ $=224$	d) 81×79 $=80^2-1^2$ $=6400-1$ $=6399$	e) 35×45 $=40^2-5^2$ $=1600-25$ $=1575$
f) 27×33 $=30^2-3^2$ $=900-9$ $=891$	g) 23×27 $=25^2-2^2$ $=225-4$ $=221$	h) 45×55 $=50^2-5^2$ $=2500-25$ $=2475$	i) 59×51 $=55^2-4^2$ $=3025-16$ $=3009$	j) 83×77 $=80^2-3^2$ $=6400-9$ $=6391$
k) 95×35 $=65^2-30^2$ $=4225-900$ $=3325$	l) 35×45 $=40^2-5^2$ $=1600-25$ $=\cancel{1975}$ 1575	m) 55×65 $=60^2-5^2$ 3600−25 $=2500-25$ 3575 $=2475$	n) 79×71 $=75^2-4^2$ $=5625-16$ $=5609$	o) 64×66 $=65^2-1^2$ $=4225-1$ $=4224$
p) 72×68 $=70^2-2^2$ $=4900-4$ $=4896$	q) 24×16 $=20^2-4^2$ $=400-16$ $=384$	r) 62×38 $=50^2-12^2$ $=2500-144$ $=\cancel{2456}$ 2356	s) 43×77 $=60^2-17^2$ $=3600-289$ $=3311$	t) 125×35 $=80^2-45^2$ $=6400-2025$ $=\cancel{4365}$ 4375

6. Use the Rule of 9's to evaluate the following:

a) 17×9 $=17(10)-17(1)$ $=170-17$ $=153$	b) 45×9 $=45(10)-45(1)$ $=450-45$ $=405$	c) 13×9 $=13(10)-13(1)$ $=130-13$ $=117$	d) 81×99 $=81(100)-81(1)$ $=8100-81$ $=8019$	e) 35×99 $=35(100)-35(1)$ $=3500-35$ 3465 $=3565$
f) 17×99 $=17(100)-17(1)$ $=1700-17$ $=1683$	g) 44×99 $=44(100)-44(1)$ $=4400-44$ $=4356$	h) 57×999 $=57(1000)-57(1)$ $=57000-57$ $=51643$ 56943	i) 123×999 $=123(1000)-123(1)$ $=123000-123$ $=122877$	j) 123×9999 $=123(10000)-123(1)$ $=1230000-123$ $=1229877$

7. Use algebra to evaluate each of the following:

a) $12(2)+22(2)-37(2)$ $=12(x)+22(x)-37(x)$ $=34(x)-37(x)$ $=-3(x)$ $=-3(2)$ $=-6$	b) $13(15)-12(15)+14(15)$ $=13x-12x+14x$ $=15x$ $=15(15)$ $=225$	c) $15(18)-9(18)+3(18)$ $=15x-9x+3x$ $=9x$ $=9(18)$ $=162$
d) $4(8)-3(24)+7(40)$ $=4x-3x+7x$ $4(8)-9(8)+35(8)$ $=8x$ $=8(8)$ $30(8)$ $=64$ $=240$	e) $3(15)-8(20)+4(25)$ $=3(3x)-8(4x)+4(5x)$ $=9x-32x+20x$ $=-3x$ $=-3(5)$ -15	f) $(17^2)+2(17)+1$ x^2+2x+1 $=17(17)+2(17)+1$ $=(x+1)^2$ $=\cancel{17(17)+1}$ $=18^2$ $=$ $=324$

8. Use the equation: $a^2-b^2=(a+b)(a-b)$ to evaluate each of the following:

a) 1000^2-999^2 $=(1000+999)(1000-999)$ $=1999(1)$ $=1999$	b) 501^2-499^2 $=(501+499)(501-499)$ $=1000(2)$ $=2000$	c) 43^2-7^2 $=(43+7)(43-7)$ $=50(36)$ $=1800$	d) 899^2-101^2 $=(899+101)(899-101)$ $=1000(798)$ $=798000$
e) 55^2-45^2 $=(55+45)(55-45)$ $=100(10)$ $=1000$	f) 355^2-145^2 $=(355+145)(355-145)$ $=500(210)$ $=105000$	g) 217^2-17^2 $=(217+17)(217-17)$ $=234(200)$ $=46800$	h) $(2^3)^2-4^3$ $=8^2-4^2$ $=(8+4)(8-4)$ $=12(4)$ $=48$

9. Use the formula $a^2 = (a+b)(a-b)+b^2$ to calculate each of the squares:

a) 99^2 $=(99+1)(99-1)+1^2$ $=100(98)+1$ $=9800+1$ $=9801$	b) 98^2 $=(98+2)(98-2)+2^2$ $=100(96)+4$ $=9600+4$ $=9604$	c) 101^2 $=(101+1)(101-1)+1^2$ $=102(100)+1$ $=10200+1$ $=10201$	d) 102^2 $=(102+2)(102-2)+2^2$ $=104(100)+4$ $=10400+4$ $=10404$	e) 51^2 $=(51+1)(51-1)+1^2$ $=52(50)+1$ $=2600+1$ $=2601$
f) 26^2 $=(26+24)(26-24)+24^2$ $=50(2)+5760$ $=100+5760$ $=5860$	g) 44×99 $=44\times(100-1)$ $=44\times 100-44\times 1$ $=4400-44$ $=4356$	h) 93×93 $=93^2$ $=(93+7)(93-7)+7^2$ $=100(86)+49$ $=8600+49$ $=8649$	i) 73×73 $=73^2$ $=(73+26)(73-26)+26^2$ $=100(47)+6860$ $=4700+6860$ $=11560$	j) 81^2 $=(81+19)(81-19)+19^2$ $=100(62)+361$ $=6200+361$ $=6561$

10. For each statement, describe a situation in which the statement is true.

a. The product of two integers equals one of the integers.

$1\times 1=1$

b. The product of two integers equals the opposite of one of the integers.

$-1\times 1=-1$

c. The product of two integers is less than both integers.

$-2\times 5=-10$

d. The product of two integers is greater than both integers.

$2\times 5=10$

11. If $a\times 23\times b = 6210$ and $a+b=N$, what is the smallest possible value of N?

$6210\div 23=270$

$270=27\times 10$

$27+10=37=N$

12. One day a sales person talked to 16 customers in 1 hour. How long would he need to work if he wanted to talk to 112 customers?

$112\div 16=7$ hours

13. Gaston withdrew $26 from his bank account each week for 17 weeks. Use integers to find the total amount Gaston withdrew over the 17 weeks. Show your work.

$\$26\times 17=\$442$

$$\begin{array}{r} 26 \\ \times 17 \\ \hline 182 \\ 26 \\ \hline 442 \end{array}$$

14. Since sunset 6 h ago, the temperature in Brandon, Manitoba, has decreased from +1 °C to -11 °C. Predict what the temperature will be 3 h from now. What assumptions did you make?

$1+(-11)=12\,°C \quad 12°C \div 6 = 2°C/hr \quad 2°C \times 3h = 6°C$

The temperature will decrease by 6°C for the next 3hrs.

15. The only possible values of x are 3, 6, 9, and 12. The only possible values of y are -10, -8, -6, and -4. What is the largest and smallest value of $x \times y$?

Largest: $x=3, y=-4 \quad 3(-4)=-12$

Smallest: $x=12, y=-10 \quad 12(-10)=-120$

16. The mean daily high temperature in Rankin Inlet, Nunavut, during one week in January was -20 °C. What might the temperatures have been on each day of the week? How many different possible answers can you find? Explain.

Example: −20°C, −23°C, −19°C, −17°C, −20°C, −21°C, −19°C

There are many different answers since this is just a prediction.

17. The mean of a group of six numbers is 40. The number 12 is removed from the group, what is the new mean?

$40\times6=240 \quad 240-12=228 \quad 228\div5=28.5$

$=45.6$

18. In the computation shown, X, Y, Z represent a different digit respectively. Determine the value of X.

$$\begin{array}{r} XY \\ \times \quad Z\,6 \\ \hline 3\,1\,2 \\ 31\,2 \\ \hline 34\,3\,2 \end{array}$$

$Z=6$, since $XY(6)=312=XY(Z)$.

$312\div6=52$

$X=5, Y=2$

19. The prime number 1999 can be written as a^2-b^2. Given that $a^2-b^2=(a+b)(a-b)$, what is the value of a^2+b^2?

$a=1000, b=999$ because $(1000+999)(1000-999)=1999(1)=1999$.

a^2+b^2

$=1000^2+999^2$

$=1000000+998001$

$=1998001$

20. The product of 119 integers is negative. At most, how many of those numbers must be negative? Explain your answer in words.

Since the product of an even number of negative integers must be positive, the total amount of negative integers must be odd. The biggest odd number between 0 and 119 is 119, so the can be 119 negative integers.

21. There are four positive numbers, a, b, c, and d (not necessarily integers). Obviously there are six ways to multiply pairs of them: ab, ac, ad, bc, bd, and cd. If I tell you that five of the six pairs of products are 2, 3, 4, 5, and 6, what is the product of the last pair.

$(2)(3)(4)(5)(6) = (ab)(ac)(ad)(bc)(bd) = (a)^3(b)^3(c)^2(d)^2$

$(ab)(ac)(ad)(bc)(bd)(cd) = (abcd)^3$ (K=24)

$(2)(3)(4)(5)(6) = 720$

$720cd = (abcd)^3$, so $720cd$ is a perfect cube (then idk)

22. Challenge: A farmer grows 3000 bananas, and wants to take them to the market to sell. The market is 1000 miles away and the only way he can get there is by a hungry camel that can carry a maximum of 1000 bananas at a time. In addition, the camel needs to eat one banana to refuel for every mile that he walks. What is the maximum number of bananas that the farmer can successfully get all the way to the market?

- Maximize the number of bananas at dropoff locations to minimize the travelling distance (and how many you can carry)

Name: Allen Xu

Date: Sep. 18

M8H 2025 Lesson 4 Order of Operations:

1. Evaluate each of the following operations. Remember the order of the operations. Show all your steps:

a) $2+5\times4$ $=2+20$ $=22$	b) $-5-8\div2$ $=-5-4$ $=-9$	c) $8\times2+6$ $=16+6$ $=22$
d) $-9\times7-20$ $=-63-20$ $=-83$	e) $8\div2+6$ $=4+6$ $=10$	f) $12-6\times(-3)$ $=12-(-18)$ $=12+18$ $=30$
g) $3+11\times4-21$ $=3+44-21$ $=47-21$ $=26$	h) $-6+24\div8-2$ $=-6+3-2$ $=-3-2$ $=-5$	i) $-12-18\div9-6$ $=-12-2-6$ $=-14-6$
j) $(7+2)\times4-5+12\div2$ $=9\times4-5+6$ $=36-5+6$ $=31+6$ $=37$	k) $11\times(3+4)-12\times2$ $=11\times7-24$ $=77-24$ $=53$	L) $-20\div(12+8)-15\div5+1$ $=-20\div20-3+1$ $=-1-3+1$ $=-4+1$ $=-3$
m) $2(12\div3+4)+12\div4$ $=2(4+4)+12\div4$ $=2(8)+12\div4$ $=16+3$ $=19$	n) $(-8\times2+12\div3)\div4\times3$ $=(-16+4)\div4\times3$ $=(-12)\div4\times3$ $=-3\times3$ $=-9$	p) $12\div4\times3\div6\times8\div4$ $=3\times3\div6\times8\div4$ $=9\div6\times8\div4$ $=1.5\times8\div4$ $=12\div4$ $=3$
Q) $(12+2)\times4-(6\times3\div2+12)$ $=14\times4-(18\div2+12)$ $=14\times4-(9+12)$ $=14\times4-21$ $=56-21$ $=35$	r) $4+(-3-2)\times(14-2)+9\times(6-2)$ $=4+(-5)\times12+9\times4$ $=4+(-60)+36$ $=-56+36$ $=-20$	s) $3(1-(5\times(5+2)+1)-2)$ $=3(1-(5\times7+1)-2)$ $=3(1-(35+1)-2)$ $=3(1-36-2$ $=3(-35-2)$ $=3(-37)$ $=-111$

2. Use BEDMAS to evaluate each of the following:

a) $4+5^2$ $=4+25$ $=29$	b) 3×2^4 $=3\times16$ $=48$	c) $11+3\times2^3$ $=11+3\times8$ $=11+24$ $=35$
d) $3\times2+3^3$ $=3\times2+27$ $=6+27$ $=33$	e) $2^2+3^2+4^2$ $=4+9+16$ $=29$	f) $4-(1+2)^2$ $=4-3^2$ $=4-9$ $=-5$
g) $2(3+4)^2-10$ $=2(7)^2-10$ $\quad 7^2=49$ $=2(14)-10$ $=28-10$ $=\cancel{18}$ $\quad 88$	h) $(4)(1+2)^2$ $=(4)(3)^2$ $=(4)(9)$ $=36$	i) $\left(\sqrt{12+4}\right)-3^2$ $=(\sqrt{16})-3^2$ $=(4)-9$ $=-5$
j) $\sqrt{3^2+4^2}$ $=\sqrt{9+16}$ $=\sqrt{25}$ $=5$	k) $3-2^3\times4$ $=3-8\times4$ $=3-24$ $\quad 3-32$ $=-21$ $\quad =-29$	L) $3^3-2^2+1^1$ $=9-4-1$ $=\cancel{4}$ $=6$
m) $5\times3^2-4$ $=5\times9-4$ $=45-4$ $=41$	n) $(-2)^2+3$ $=(4)+3$ $=7$	p) -2^2+6 $=4+6$ $\quad -2^2=-4$ $=10$ $\quad (-2)^2=4$ NO BRACKETS
Q) $40\div2\times3^2-4$ $=40\div2\times9-4$ $=20\times9-4$ $=180-4$ $=176$	r) $\dfrac{3^3-2^2+5}{12\div3}$ $=\dfrac{27-4+5}{4}$ $=\dfrac{28}{4}$ $=7$	s) $\dfrac{(21-17)\div3}{10^2\div20}$ $=\dfrac{4\div3}{100\div20}$ $=\dfrac{\frac{4}{3}}{\frac{100}{20}}$ $=\frac{4}{3}\div\frac{5}{1}$

$=\frac{4}{3}\times\frac{1}{5}$

$=4$

t) $2\times(14\div2)^2+5\times12$ $=2\times(7)^2+5\times12$ $=2\times(14)+5\times12$ $=28+60$ $=88$ $7^2=49$ $=158$	u) $4\times(13+8)-8^2\div(2\times4)$ $=4\times(21)-64\div8$ $=84-8$ $=76$	v) $(34+12)\times8\div2+2^5$ $=46\times8\div2+25$ $=368\div2+25$ $=184+25$ $=\cancel{209}$ $=216$ $2^5=32$ $2^5\neq25$
w) $36\div(6+3)\times(3^3+17)\div4$ $=36\div9\times(9+17)\div4$ $=36\div9\times26\div4$ $=4\times26\div4$ $=104\div4$ $=\cancel{26}$ 44 $3^3=27$ not 9	x) $(3^4\div9)+32-(5\times10)+6$ $=(81\div9)+32-(50)+6$ $=9+32-50+6$ $=41-50+6$ $=-9+6$ $=-3$	y) $18+(57-38)\times10+4^2$ $=18+19\times10+16$ $=18+190+16$ $=208+16$ $=224$

3. What operations can be placed into the boxes so that the expression will be true:

a) $12\ \boxed{+}\ 2\ \boxed{+}\ 3\ \boxed{\times}\ 1=17$	b) $9\ \boxed{-}\ 14\ \boxed{\div}\ 2\ \boxed{\times}\ 3=-12$	c) $9\ \boxed{\times}\ 4\ \boxed{-}\ 44\ \boxed{\div}\ 2=14$

4. Indicate the all mistakes in each of the following examples shown below. There are at least one mistake in each example:

a) Julie's Work: Did 3+2 first $15\div(3+2)-7\times4$ $=15\div(5)-28$ $=3-28$ $=-25$	b) Tim's work: Did 2-6 first $-27+9\times(2-6)$ $=-27+9\times(-4)$ $=-27+(-36)$ $=63$	c) Tracy's Work: Might be $-3-4-5\times2+2\div(-1)$ $=-7-10+-2$ $=-15$ $-7-10+-2$ $=-7-10-2$ $=-19$

5. Where can you insert a pair of brackets into the following expression so that the value can be maximized?

$$3+(6\times9+2-5)\times4$$

$=3+(54+2-5)\times4$
$=3+51\times4$
$=3+204$
$=207$

6. Jason wrote six math exams and got the following scores: 87%, 74%, 65%, 92%, 78%, and 99%. What is the average score for his six exams?

$(87+74+65+92+78+99)\div6$
$=495\div6$
$=\cancel{63}$
$=82.5\%$

7. Thomas plays on the school basketball team. He gets 3 points if he scores a 3pointer, 2 points for fields-goals, and 1 point for each foul shot made. He scored 4 3pointers, 6 fg, and 9 free throws. How many points did he get?

$(4\times3)+(6\times2)+(9\times1)$

$=12+12+9$

$=33.$

8. A certain small factory employs 98 workers. Of these 10 receive a wage of \$200 per day and the rest receive \$100 per day. To the management, a week is equal to 6 working days. How much does the factory pay out for each week?

$6(10(200)+(98-10)(100))$

$=6(2000+88(100))$

$=6(2000+8800)$

$=6(10800)$

$=64800$

9. The final grade in a course is the average of the scores on 10 tests. Each test is graded on a scale of zero to 100 inclusive. A student's average on the first 7 tests was 84. The final grade of the student in the course was 63. What was the average student grade on the last 3 tests?

$(10(63)-7(84))\div3$

$=(630-588)\div3$

$=42\div3$

$=14$

10. Use numbers 1, 2, 3, and 4, each once to replace variables in $a+b\times c^d$ ($1+2\times3^4$). What is the maximum value of the expression?

$1+2\times3^4$

$=1+2\times81$

$=1+162$

$=163$

11. Challenge: How many digits are in the value of $2^{2003}\times5^{1952}\div4^{27}$?

① $2\times5=1\underline{0}$

② $2^2\times5^2=1\underline{00}$

③ $2^3\times5^3=1\underline{000}$

$4^{27}=(2^2)^{27}=2^{54}$

$\dfrac{2^{2003}\times5^{1952}}{2^{54}}$

$=2^{1949}\times5^{1952}$

$=2^{1949}\times5^{1949}\times5^3$

$=10^{1949}\times125$

$=\underbrace{125}_{3\text{ digits}}\,\underbrace{0000\ldots}_{1949\text{ zeros}}$

$1949+3=1952$ digits

Name: Allen Xu
Date: Sep. 22

Math8H 2025 Lesson 5 Divisibility Rules

1. How many of the following numbers are divisible by 3? (No calculators)

a) 115	b) 285	c) 498	d)9381	e) 3951	f) 52376
No	Yes	Yes	No Yes	Yes	No

2. How many of the following numbers are divisible by 11? (No calculators)

a) 4013	b) 4301	c) 30932	d)7392	e)69319	f)495614
No	Yes	Yes	Yes	No	No

3. How many of the following numbers are divisible by 7? (No calculators)

a)1645	b) 4398	c)23030	d)46231	e)18557	f)82311
Yes	No	Yes	No	Yes	No

4. Given that the following numbers are all divisible by 3, what are the values of "A"?

a) 4A3	b) 3981A	c)392AA	d)29A314A
2, 5, 8	0, 3, 6, 9	2, 5, 8	1, 4, 7

5. Given that the following numbers are all divisible by 11, what are the values of "A"?

a) 6A2	b) 1234A	c)356A2A	d) 356AA
8	2	3, ~~8~~ 11 − (5 + 2A) = 0 22 = (5 + 2A) 33 = 5 + 2A) 28 =	Impossible

6. Indicate if the following statements are TRUE or FALSE:

a) If a number is divisible by 9, then it must be divisible by 3

True

b) If a number is divisible by 3, then it must be divisible by 9

False

c) If a number is divisible by 2 and 4, then it must be divisible by 8

True

d) All even numbers that are divisible by 3 are also divisible by 6

True

e) If a number is divisible by 5, then the last digit must be a 0

False

f) The number 3AA78 can never be divisible by 11

True

g) If "A" is divisible by 3 and "B" is divisible by 3, then A+B is also divisible by 3

True

7. If the 5-digit number $1\cancel{7}3\cancel{2}p$ is divisible by 9, determine the value of p.

$p=5$

8. What digit can replace K so that the number $9K73K0$ is divisible by 6?

1, 4, 7

9. Suppose the 6 digit number 2A5A93 is divisible by both 3 and 11, what are the possible values of the single digit number "A"?

A=1

10. What is the smallest positive integer that is divisible by $\cancel{2}$, 3, 4, 5, and $\cancel{6}$?

You check 2 and 3 for multiple of 6, so 6 is cancelled out.
There's already 4, so 2 is cancelled out.
$3\times4\times5=60$

11. A boy can divide his marble collection into even groups of 3, 4, or 6. What is the smallest number of marbles in his collection?

$3\times4=12$

ends with 0
$\cancel{2}, 3, \cancel{5}$ $\cancel{4}, \cancel{6}, 8$

12. What is the smallest 3 digit number that is divisible by the first 3 prime as well as the first 3 composite numbers?

ends with 0
first two digits' sum must be a multiple of 3. Try 120
1 2 0 factors of 120: 1, 120, 2, 60, 3, 40, 4, 30, 5, 24, 6, 20, 8, 15, 10, 12

13. The number $3N+63$ is divisible by 7. Explain whether N would be divisible by 7.

If N=7, it would be divisible by 7 because 3(7)+9(7)=12(7)

14. Use the digits 4, 5, 7, 9, and one additional digit, construct the largest possible 5-digit number divisible by 6.

98754 4+5+7+9=25 25+8=33 divisible by 3
Put the 4 at the end divisible by 2

3, $\cancel{4}$, $\cancel{5}$, $\cancel{2}$, $\cancel{10}$

15. Find the least perfect square number which is divisible by each of the numbers 8, 12, 15 and 20

Perfect squares that end with 0: 10^2, 20^2, 30^2 ... Since they end with two 0s, 4, 5, 2, and 10 are solved
30^2 is the answer because it's divisible by 3 N=3600

16. It is given that a number is divisible by both 6 and 26. Name two other factors of the number. Show your work.

$6 \times 26 = 156 \quad 156 = 2 \times 78$

Two other factors of the number can be 2 and 78.

$N = 6 \times A = 2 \times 3 \times A$

$N = 26 \times B = 2 \times 13 \times B$

$N = 2 \times 3 \times 13 \times \boxed{C}$

17. The integers a and b are both divisible by 2. Determine and explain whether each of the following statements would be always true or not. Provide a counter example to prove that a statement may not always be true. [Hint: If you are stuck, consider plugging in numbers for a and b and see if you can determine a trend.]

a. $a+b$ is divisible by 2 True

$a = \frac{a}{2}(2) \quad \frac{a}{2}(2) + \frac{b}{2}(2) = \frac{a+b}{2}(2)$

$b = \frac{b}{2}(2)$

b. $a-b$ is divisible by 2 True

$a = \frac{a}{2}(2) \quad \frac{a}{2}(2) - \frac{b}{2}(2) = \frac{a-b}{2}(2)$

$b = \frac{b}{2}(2)$

c. $a+b$ is divisible by 4 False

Example: $6+8=14$

$14 \div 4 = 3.5$

d. a^2+b^2 is divisible by 4 True

$a^2 = \frac{a}{2}(2)^2 = \frac{a}{2}(4)$

$b^2 = \frac{b}{2}(2)^2 = \frac{b}{2}(4)$

e. ab is divisible by 4

$a = \frac{a}{2}(2) \quad ab = \frac{a}{2}(2) \times$

$b = \frac{b}{2}(2)$

Challenge Section:

18. When Rachel divides her favourite number by 7, the remainder is 5. What will the remainder be if Rachel multiply her favourite number by 5 then divide by 7?

fav num $= 7k+5$

new fav num $= 5(7k+5) = 35k+25$

$35k \div 7 + 25 \div 7 =$

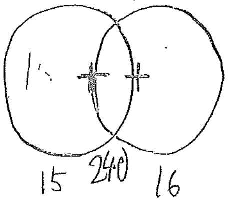
$= 5k+3$ with a remainder of 4

19. The integers r, s, and t are three consecutive integers. Their sum is always divisible by at least 2 integers. What are those two numbers?

$r = s-1 \quad t = s+1 \quad r+s+t = (s-1)+(s)+(s+1) = 3s$

The two numbers are 3 and 1

20. How many of the integers between 1400 and 2400, inclusive are an integer multiple of either 15 or 16 (or both)?



find 15: $\frac{1400}{15}=93.\dot{3}$ $\frac{2400}{15}=160$ $160-94+1=67$

= answer

find 16: $\frac{1400}{16}=87.5$ $\frac{2400}{16}=150$ $150-88+1=63$

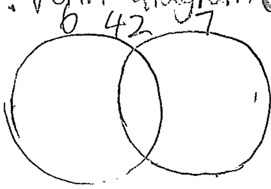
find 240: $\frac{1400}{240}=5.8\dot{3}$ $\frac{2400}{240}=10$ $10-6+1=5$

$67+63-5=125$

21. How many numbers between 200 and 2000 are divisible by 6 or 7 but not both?

To solve anything like this (How many numbers are divisible by...)

1. Venn diagram (find the ones divisible by num 1 or num 2 but not both)



2. Multiples of 6: $\frac{200}{6}=33.\dot{3}$, $\frac{2000}{6}=333.\dot{3}$

~~204 (6·34), 204 (6·…), 204 (6·…)~~

3. Do the same for 7 [287]

4. Do the same for 42 [43]

Number line:

33.3×6. 34×6 — 1st num

333×6 333.3×6 — Last num

Sum of num: $333-33+1=$ ~~301~~

Because include the first num

5. $(301-43)+(287-43)=502$ Answer!

Multiples of 6 without multiples of 42

Multiples of 7 without multiples of 42

Answer $N=471$.

22. Ultimate Challenge: The digits 1, 2, 3, 4, and 5 are each used once to compose a five digit number *abcde* such that the three digit number *abc* is divisible by 4, *bcd* is divisible by 5, and *cde* is divisible by 3. Find the digit "a"

$d=5$

c or e = 3

c must be even

so $c=2$ or 4

$e=3$

$c=4$ because cde is divisible by 3

1	2	4	5	3
a	b	c	d	e

$a=1$!

Name: Allen Xu Date:

Math 8 Honours HW Lesson 6 Prime, Factors, LCM, and GCF

1. What does it mean that two values are relatively prime? Explain?

If the GCF of two values is 1, they are relatively prime.

2. If the GCF of "a" and "b" is one of the values "a" or "b", then what is the relationship between "a" and 'b'?

a is one of b's factor

3. IF the LCM of "a" and "b" is one of the values "a" or "b", then what is the relationship between "a" and 'b'?

a is b's multiple

4. Circle all the prime numbers below. If it's not a prime number, give one of its factors other than 1:

23	29	39 3	43	49 7	61
71	93 3	79	101	109	113
117 13	119 7	137	147 3	157	169 13

5. Given each set of numbers below, find the greatest common factor (GCF) and lowest common multiple (LCM):

a) $\langle 15,24\rangle$ $15=3^1\times5^1$ $24=2^3\times3^1$ GCF: 3 LCM: 120	b) $\langle 18,12\rangle$ $18=2^1\times3^2$ $12=2^2\times3^1$ GCF: 6 LCM: 36	c) $\langle 16,8\rangle$ $16=2^4$ $8=2^3$ GCF: 8 LCM: 16
d) $\langle 35,14\rangle$ $35=5^1\times7^1$ $14=2^1\times7^1$ GCF: 7 LCM: 70	e) $\langle 65,91\rangle$ $65=5^1\times13^1$ $91=7^1\times13^1$ GCF: 13 LCM: 455	f) $\langle 195,221\rangle$ $195=3^1\times5^1\times13^1$ $221=13^1\times17^1$ GCF: 13 LCM: 3315

How many prime numbers are there less than 100? (list them out)

25 of them are prime:

2, 3, 5, 7, 11, 13, 17, 19, 21, 23, 29, 31, 37, 41, 43, 47,

7. Is "1" a prime number? Explain:

No, because it only has one factor, but prime numbers have two factors.

8. Check if each of the following integers are prime numbers: (Use the Prime Number TEST). If the number is NOT a prime, state of it's prime (other than 1)

a) 117 $\sqrt{117} \approx 10$ 2, 3, 5, 7. $117 \div 3 = 39$	b) 167 $\sqrt{167} \approx 12$ 2, 3, 5, 7, 11. 167 is prime	c) 279 $\sqrt{279} \approx 16$ 2, 3, 5, 7, 11, 13 $279 \div 3 = 93$
d) 913 $\sqrt{913} \approx 30$ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 $913 \div 11 = 83$	e) 493 $\sqrt{493} \approx 22$ 2, 3, 5, 7, 11, 13, 17, 19 $493 \div 17 = 29$	f) 891 $\sqrt{891} \approx 29$ 2, 3, 5, 7, 11, 13, 17, 19, 23 $891 \div 3 = 297$
g) 1003 $\sqrt{1003} \approx 31$ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 $1003 \div 17 = 59$	h) 451 $\sqrt{451} \approx 21$ 2, 3, 5, 7, 11, 13, 17, 19 $451 \div 11 = 41$	i) 717 $\sqrt{717} \approx 26$ 2, 3, 5, 7, 11, 13, 17, 23 $717 \div 3 = 239$
j) 637 $\sqrt{637} \approx 25$ 2, 3, 5, 7, 11, 13, 17, 19, 23 $637 \div 7 = 91$	k) 217 $\sqrt{217} \approx 14$ 2, 3, 5, 7, 11, 13 $217 \div 7 = 31$	L)323 $\sqrt{323} \approx 17$ 2, 3, 5, 7, 11, 13, 17 $323 \div 17 = 19$

9. Suppose "A" and "B" are integers with a GCF of "x" and a product of "y". What is the LCM in terms of "x" and "y"?
AB

$[x, y] = y$

10. Suppose 1316 and 2820 have a GCF of 188. What is the LCM?
$1316 \times 2820 \div 188 = 19740$

11. What is the smallest number with four factors?

6 has 1, 2, 3, and 6

12. Find a number between 1 to 100 that has 5 factors?

16 because it's a perfect square

13. Why do perfect squares have an "odd" number of factors? Explain:

because they have one pair of repetitive factors.

14. What number less than 100 has the greatest number of factors?

15. In the multiplication shown below, each letter represents a different digit. What digit does the letter "C" represent?

$$\begin{array}{r} ABCDE \\ \times E \\ \hline EDADE \end{array}$$

E = ~~1~~, 5, or 6

can't be 1 because the result is different

16. Find the smallest two-digit number that is twice the product of its digits

17. What is the product of the LCM and GCF of two distinct numbers "a" and "b"?

ab

18. Suppose that $N_1 = a^3 \times b^4 \times c^5$ and $N_2 = a^2 \times b^1 \times c^6 \times d^2$, where "a", "b", "c" and "d" are all prime factors. What is the GCF and LCM in terms of "a", "b", "c" and "d"?

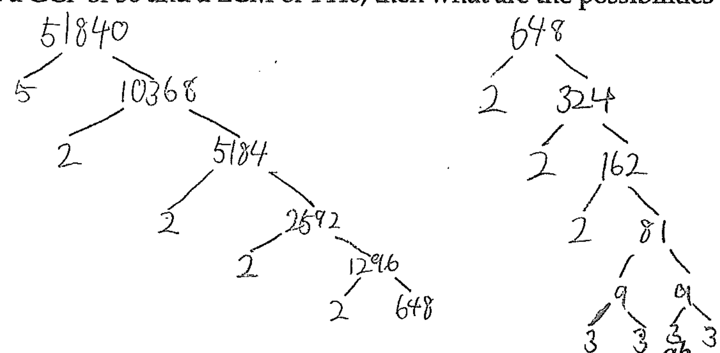
$(N_1, N_2) = a^2 \cdot b^1 \cdot c^5$

$[N_1, N_2] = a^3 \cdot b^4 \cdot c^6 \cdot d^2$

19. If two numbers "a" and "b" have a GCF of 36 and a LCM of 1440, then what are the possibilities of "a" and "b" if a > b?

$1440 \times 36 = 51840$

$51840 = 2^7 \times 3^4 \times 5^1$



20. Challenge: Given that a, b, c, d, e, f, g, h, and i all represent a different digit from 1 to 9. If $\frac{ab}{cde} + \frac{fg}{hi} = 7$, then what numbers do each letter represent?

Name: Allen Xu period 7 Date: Oct. 9, 2025

Math 8H Lesson 7 Prime Factorizations (2025)

1. What is the purpose of finding the prime factorization of a number?
   Find GCF, LCM, number of factors, etc.

2. When given a number "N" in the form of its prime factorization, how do you find the number of factors of "N"? ie: $N = 2^a \times 3^b \times 5^c$
   $(a+1)\times(b+1)\times(c+1)$

3. When given the number "N", with $N = 2^3 \times 6^4$, a student got the number of factors of 'n' as $4\times5=20$. What did this student do wrong? Explain:
   6 is not prime, $6^4 = 2^4$ and 3^4

4. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are perfect squares? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:
   Only use the exponents that are even because odd exponents are not perfect squares

   $N=2^8\times3^9\times5^{10}$

2^0	3^0	5^0
2^2	3^2	5^2
2^4	3^4	5^4
2^6	3^6	5^6
2^8	3^8	5^8
		5^{10}

   $5\times5\times6=150$

5. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are ODD numbers? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:
   You ignore all the even bases because all the multiples of 2 are even

6. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are EVEN numbers? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:
   You use the even and odd bases except 2^0 because $2^0=1$ which is odd

7. When given a number "N" in the form of its prime factorization, how do you find the SUM of all the factors of "N"? Ie: $N = 2^4 \times 3^5 \times 5^6$ .Explain:
   $(2^0+2^1+2^2+2^3+2^4)(3^0+3^1+3^2+3^3+3^4+3^5)(5^0+5^1+5^2+5^3+5^4+5^5+5^6)$
   Because everything in the brackets pair up to make all the factors

8. When given TWO or more numbers in their prime factorization form, how would you find their GCF? Explain: ie: $N_1 = 2^3 \times 3 \times 5^4$ $N_2 = 2^4 \times 3^2 \times 5^2$ $N_3 = 2 \times 3^4 \times 5^6$
   Use the lowest exponents of the bases that all numbers share, because factors are always SMALLER!

9. When given TWO or more numbers in their prime factorization form, how would you find their LCM? Explain: ie: $N_1 = 2^3 \times 3 \times 5^4$ $N_2 = 2^4 \times 3^2 \times 5^2$ $N_3 = 2 \times 3^4 \times 5^6$
   Use the largest exponents of all the bases, because multiples are always BIGGER!

10. Prime the Prime Factorization of each number below. Then indicate whether if it is a perfect square or perfect cube, neither, or both:

24 $24 = 2^3 \times 3^1$ neither	800 $800 = 2^5 \times 5^2$ neither	864 $864 = 2^5 \times 3^3$
1800 $1800 = 2^3 \times 5^2 \times 11^1$ neither	648 $648 = 2^3 \times 3^4$ neither	210 $210 = 2 \times 3 \times 5 \times 7$ neither
5040 $5040 = 2^4 \times 3^2 \times 5^1 \times 7^1$ neither	3136 $3136 = 2^6 \times 7^2$ perfect square	2744 $2744 = 2^3 \times 7^3$ perfect cube
$N = 2^2 \times 50 \times 5$ $N = 2^3 \times 5^3$ perfect cube	$N = 64 \times 25 \times 49$ $N = 2^6 \times 5^2 \times 7^2$ perfect square	$N = 30 \times 45 \times 40$ $N = 2^4 \times 3^3 \times 5^3$ neither

11. Given each pair of numbers in their prime factorization, find the GCF and LCM

25 & 45 $25 = 5^2$ $45 = 3^2 \times 5^1$ $(25, 45) = 5$ $[25, 45] = 225$	$N_1 = 2^2 \times 3^3$ & $N_2 = 2^3 \times 5^2$ $(N_1, N_2) = 4$ $[N_1, N_2] = 5400$
$N_1 = 2^3 \times 5 \times 7$ & $N_2 = 2 \times 3^4 \times 5^2$ $(N_1, N_2) = 10$ $[N_1, N_2] = 113400$	$N_1 = 2^{\cancel{3}\,6} \times \cancel{4} \times \cancel{6}\,3^1$ & $N_2 = \cancel{10^2} \times \cancel{8}$ $2^5 \times 5^2$ $(N_1, N_2) = 32$ $[N_1, N_2] = 4800$

$N_1 = a^2b^{13}c^{15}$ & $N_2 = a^5b^8c^{11}d^5$ $(N_1, N_2) = a^2 \times b^8 \times c^{11}$ $[N_1, N_2] = a^5 \times b^{13} \times c^{15} \times d^5$	$N_1 = 2^7$ & $N_2 = 3^5$ $(N_1, N_2) = 1$ $[N_1, N_2] = 31104$

12. Use the prime factorization to find the number of factors:

$2^4 \times 3^2 \times 5^2 =$ $\left(\frac{2^4-1}{2-1}\right)\left(\frac{3^2-1}{3-1}\right)\left(\frac{5^2-1}{5-1}\right)$ too lazy to calculate	$3^4 \times 5^3 \times 11^8 =$ $\left(\frac{3^4-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)\left(\frac{11^8-1}{11-1}\right)$
20124 $20124 = 2^2 \times 3^2 \times 13^1 \times 43^1$ $\left(\frac{2^2-1}{2-1}\right)\left(\frac{3^2-1}{3-1}\right)(13)(43)$	4500 $4500 = 2^2 \times 3^2 \times 5^3$ $\left(\frac{2^2-1}{2-1}\right)\left(\frac{3^2-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)$
$2^6 \times 3^6 \times \cancel{36} =$ $\left(\frac{2^6-1}{2-1}\right)\left(\frac{3^6-1}{3-1}\right)$	$3^6 \times 7^{12} \times \cancel{21}$ $\left(\frac{3^6-1}{3-1}\right)\left(\frac{7^{12}-1}{7-1}\right)$
$N = \cancel{8^2}\,2^6 \times 3^6 \times \cancel{15^2}\,5^2$ $\left(\frac{2^6-1}{2-1}\right)\left(\frac{3^6-1}{3-1}\right)\left(\frac{5^2-1}{5-1}\right)$	$2^6 \times 3^3 \times 2^6 \times 5^3 = 2^{12} \times 3^3 \times 5^3$ $N = 12^3 \times 20^3$ $\left(\frac{2^{12}-1}{2-1}\right)\left(\frac{3^3-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)$

13. How do you tell if a number is a perfect square or cube by looking at the prime factorization:

See if all the exponents are multiples of 2/3

14. Find the lowest value of N such that the square root will become a positive integer:

a) $\sqrt{2^3 5^1 7^2 N}$

b) $\sqrt{4^2 7^2 5^2 N}$ 2^4

c) $\sqrt{3^4 5^3 12 N} = \sqrt{3^5 5^3 2^2 N}$ $2^2 \times 3^1$

N=10

N=1

N=15

d) $\sqrt{38412N}$

$38412 = 2^2 \times 3^2 \times 1067^1$

N=1067

e) $\sqrt{13992N}$

$13992 = 2^3 \times 3^1 \times 11^1 \times 53^1$

$N = 3 \times 11 \times 53$

N=3498

$N = 1749 \times 2$

f) $\sqrt{664(N-1)}$

$664 = 2^3 \times 83^1$

$N = 2 \times 83 + 1$

N=167

15. Find the lowest value of N such that the integer will have the indicated the indicated number of factors:

a) $2^3 3^N$ (8 *factors*)

N=1

b) $(8) \times 27N$ (48 factors)

$8 = 2^3$

$27 = 3^3$

$(3+1)(3+1) = 16$

$48 \div 16 = 3$

$25 = 5^2$

N=25

c) $2^3 3^4 N^2$ (56 *factors*)

$(3+1)(4+1)(2+1)$

$= (4)(5)(3)$

$= 60$

$12^2 = 3^2 2^4$

$2^3 3^4 12^2 = 2^7 3^6$

$(7+1)(6+1) = 56$

N=12

16. Two positive integers have a GCF of $2 \times 3 \times 5$ and a LCM of $2^3 \times 3^4 \times 5 \times 7$. If one of the numbers is 210, find the other number.

$210 = 2 \times 3 \times 5 \times 7$

The other number must have: $2^3, 3^4, 5$.

So the other number is $2^3 \times 3^4 \times 5$

17. Find the smallest number N, such that $2^3 3^4 N^2$ has 56 factors.

? That's the same thing as 15.c)

18. Two numbers are "*relatively prime*" if they do not share any common factors other than 1. How many positive integers less than or equal to 40 are relatively prime to 40?

$40 = 2^3 \times 5^1$ Only numbers without 2 or 5 as their factor

1 ~~2~~ 3 ~~4~~ ~~5~~ ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~

21 ~~22~~ 23 ~~24~~ ~~25~~ ~~26~~ 27 ~~28~~ 29 ~~30~~ 31 ~~32~~ 33 ~~34~~ ~~35~~ ~~36~~ 37 ~~38~~ 39 ~~40~~

19. Challenge: Suppose there are 1000 lockers and 1000 people. The first person opens all the lockers; the second person closes every second locker; the third person changes the state of every third locker [ie: if it's open, he closes it or if it's closed, he opens it]. This process continues, where the nth person changes the state of every nth locker. After all 1000 people have gone through, how many lockers are open?

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2	O	X	O	X	O	X	O	X	O	X	O	X	O	X	O	X	O	X	O	X
3	O	X	X	X	O	O	O	X	X	X	O	O	O	X	X	X	O	O	O	X
4	O	X	X	O	O	O	O	O	X	X	O	X	O	X	X	O	O	O	O	O
5	O	X	X	O	X	O	O	O	X	O	O	X	O	X	O	O	O	O	O	X
6	O	X	X	O	X	X	O	O	X	O	O	O	O	X	O	O	O	X	O	X
7	O	X	X	O	X	X	X	O	X	O	O	O	O	O	O	O	O	X	O	X
8	O	X	X	O	X	X	X	X	X	O	O	O	O	O	O	X	O	X	O	X
9	O	X	X	O	X	X	X	X	O	O	O	O	O	O	O	X	O	O	O	X
	O			O					O											

Perfect Squares → just calculate all the perfect squares bc they have an odd amount of factors,